THERMAL REGIME OF THE OPTICAL ELEMENTS OF THE LIGHTING SYSTEM OF A SOLID-STATE LASER WITH NATURAL COOLING

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The authors obtained approximate relationships for calculating the non-steady-state temperatures of the optical elements of the lighting system. Calculations were applied in the investigation of the effect of various factors on the thermal regime of lasers.

In the energy balance of pumping for a solid-state laser, thermal losses represent a substantial proportion. The efficiency of a laser is usually equal to $\sim$1%, and the remaining pumping energy, except for the losses in the discharge circuit, is released in the form of heat in the elements of the lighting system, i.e., the pumping lamp, the active element, and the illuminator [1]. Under the conditions of natural cooling, the energy exchange between individual elements of the system leads to complex heat-exchange conditions on their boundaries.

For practical evaluations, the most important information is that on the temperature distribution in the active element. For approximate calculations of the temperature field and the thermal deformations of the active element we may use the relationships obtained as a result of a number of substantial simplifications and the stipulation of the basic and most characteristic ways of heat transfer in the lighting system [2, 3]. However, there are not sufficient calculations for analyzing the effect of different parameters on the thermal regime of the instrument as a whole.

The fullest possible information on the thermal regime of a laser can be obtained on the basis of a calculation of the thermal field of a system of bodies, viz., the optical elements of the lighting system. The accurate solution of such a problem can be realized only on a computer.

However, for practical calculations it is indispensable to have an approximate analytical solution that is simple in form and ensures an accuracy sufficient for engineering purposes.

The use of approximate calculations is all the more justified in the present case because in systems of bodies such as the lighting systems of lasers, the regularities of heat exchange have not yet been sufficiently studied, and we lack proven methods of calculating thermal conductivities as well as accurate data on the magnitudes of heat release in the optical elements. Approximate theoretical relationships make it possible to determine the criticality of the calculated temperatures to a change in the magnitude of conductivities and heat releases, and thus to specify the requirements that the accuracy of determining these parameters has to satisfy.

In most cases the thermal model of the lighting system of a solid-state laser reduces to a system of three bodies: the parallelly arranged cylinders with the heat source (lamp and the active element) in the joint heat-releasing cylindrical shell (illuminator).

Arrangements of such systems are presented in [2, 3]. In [4] it was shown that it is expedient to solve this problem in two stages, assuming at the first stage that the temperature field of the bodies of the system is homogeneous. In addition to that, we will confine ourselves to examining the case of quasicontinuous heat release in periodic-pulse operation of the laser. Under conditions of natural cooling, such an assumption leads to an error not exceeding 5% with recurrence frequencies of more than 0.05 Hz [2]. We introduce another assumption on the basis of the use of existing methods of classifying thermal models of

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systems of bodies with sources of energy [5, 6]. In the mathematical description of the problem we adopt the assumptions explained in [6]: the thermal connections between the bodies have no thermal capacity, and the thermophysical and heat-exchange coefficients do not depend on the temperature. We introduce the assumption that there are no thermal connections with the environment for bodies surrounded by a shell, i.e., lamp and active element. In reality these bodies take part in direct heat exchange with the environment through the end-face sections of the surface; this causes a temperature gradient in the axial direction that is symmetrical with respect to the central section in the middle between the end faces [2].

When the ratio of the length of the element L to the diameter d is sufficiently large, the heat removal has only a weak effect on the temperature level in the central section. For real elements of the lighting system $L/d \geq 10$, and neglecting the axial heat removal leads to a solution that corresponds to the conditions of heat exchange for the central section. The temperatures thus calculated correspond to the maximum temperatures of the elements. Within the framework of the adopted assumptions, the mathematical statement of the problem reduces to the integration of a system of three linear ordinary differential equations of first order:

$$\left.\begin{aligned} &C_1 \frac{d\vartheta_1}{d\tau} + (\sigma_{12} + \sigma_{13})\,\vartheta_1 - \sigma_{12}\vartheta_2 - \sigma_{13}\vartheta_3 = P_1, \quad \text{(a)} \\ &C_2 \frac{d\vartheta_2}{d\tau} + (\sigma_{21} + \sigma_{23})\,\vartheta_2 - \sigma_{21}\vartheta_1 - \sigma_{23}\vartheta_3 = P_2, \quad \text{(b)} \\ &C_3 \frac{d\vartheta_3}{d\tau} + (\sigma_{31} + \sigma_{32} + \sigma_{3c})\,\vartheta_3 - \sigma_{31}\vartheta_1 - \sigma_{32}\vartheta_2 = P_3. \quad \text{(c)} \end{aligned}\right\} \tag{1}$$

The solution of system (1) can be greatly simplified by introducing the effective body $E = 1 \cup 2$ [5, 6]. The results of numerical evaluations show that in real lighting systems we find reduced thermal connections characterized by the parameter $\varkappa_\Sigma = \sigma_{3E}/\sigma_{3c}$ (the criterion of moderateness $0.04 < \varkappa_\Sigma < 2$) [5] at which the reduction of the system of bodies to the thermal model of two bodies (the examined one and the effective one) proves to be very effective [5, 6]. It should be pointed out that the examined body in the given system is a shell for the effective body, and in steady-state thermal regime the introduction of the body $E = 1 \cup 2$ does not introduce errors into the determination of the value of ϑ_3 since the sum of the thermal flows $P_E = P_1 + P_2$ is one way or another scattered over the shell. We also point out that the introduction of the effective body favors the condition $C_1/C_2 \approx 1$ [5, 6] realized in most types of lighting systems [2].

In introducing the effective body we assume that

$$\vartheta_E\big|_{\tau=0} = \sum \frac{\sigma_{ji}}{\sigma_{jE}}\,\vartheta_i^{(H)} = \frac{\sigma_{31}\vartheta_1^{(H)} + \sigma_{32}\vartheta_2^{(H)}}{\sigma_{3E}} = \vartheta_E^{(H)} = 0,$$

$$\sigma_{jE} = \sigma_{3E} = \sigma_{31} + \sigma_{32}, \quad \sigma_{Ec} = \sigma_{1c} + \sigma_{2c} = 0, \quad C_E = C_1 + C_2.$$

As a result, (1) reduces to a system of two equations:

$$\left.\begin{aligned} &\frac{1}{m_3}\frac{d\vartheta_3}{d\tau} + (\varkappa_\Sigma + 1)\,\vartheta_3 - \varkappa_\Sigma\vartheta_E = \frac{P_3}{\sigma_{3c}}, \quad \text{(a)} \\ &\frac{1}{m_3}\frac{d\vartheta_E}{d\tau} + \frac{\varkappa_\Sigma}{C_E'}\vartheta_E - \frac{\varkappa_\Sigma}{C_E'}\vartheta_3 = \frac{P_E}{\sigma_{3c}}\frac{1}{C_E'}, \quad \text{(b)} \end{aligned}\right\} \tag{2}$$

where $m_3 = \sigma_{3c}/C_3$; $C_E' = C_E/C_3$.

Further calculations can be substantially simplified by the introduction of two additional assumptions.

Assumption 1. The illuminator (body 3) is heated much more slowly than the lamp and the active element (effective body), and up to and including the steady-state value of ϑ_3, the magnitude of ϑ_3 changes little. Such an assumption is justified for $C_E' \ll 1$, which is usually realized in solid-state lasers. In accordance with the data of experimental verification, the lamp and the active element without shell are heated to the steady-state temperature level with an accuracy of ~10% within ~3 min, and when placed in the illuminator,

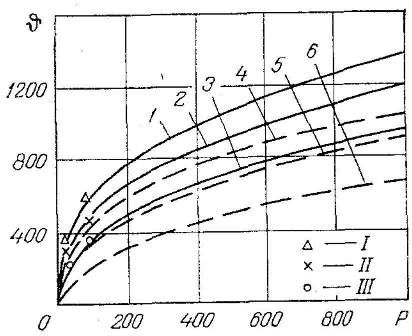

Fig. 1

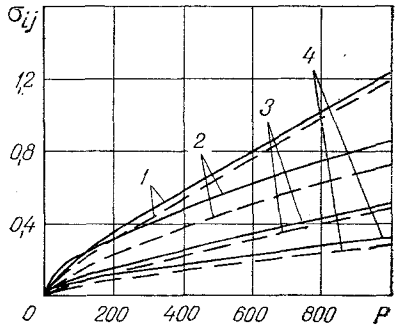

Fig. 2

Fig. 1. Dependence of the steady-state mean-surface superheating of the optical elements ϑ_i (K) on the mean pumping power P (W). Calculation: 1) superheating of the lamp ϑ_1; 2) superheating of the active element ϑ_2; 3) superheating of the illuminator ϑ_3; 4) ϑ_2 for $\sigma_{12}=0$; 5) ϑ_2 for $\vartheta_3=0$; 6) ϑ_2 for $\sigma_{12}=0$ and $\vartheta_3=0$. Experiment: I) ϑ_1; II) ϑ_2; III) ϑ_3.

Fig. 2. Dependence of the thermal conductivities σ_{ij} (W/°K) on the mean pumping power P (W): 1) thermal conductivity from the lamp to the illuminator σ_{13}; 2) thermal conductivity from the illuminator to the environment σ_{3C}; 3) thermal conductivity from the active element to the illuminator σ_{23}; 4) thermal conductivity from the lamp to the active element σ_{12}; dashed lines represent the radiant components of the corresponding values of σ_{ij}.

within ∿20 min. The heating of the effective body in the shell is characterized by two stages: at the first stage the heating rate is characterized by the inherent thermal inertia of the body E, and at the second stage the slow increase in the temperature of the body E is due to the gradual heating of the shell. With the given assumption, Eq. (2b) can be solved independently of (2a) on the assumption that $\vartheta_3=\text{const}$. After substitution of the value ϑ_E obtained in this way into (2a), Eq. (2a) can be solved with respect to ϑ_3 for $\vartheta_E=\text{const}$. Then the substitution of ϑ_3 into (1a, b) and the use of the given assumption concerning the slow heating of the illuminator make it possible to obtain approximate relationships for ϑ_1 and ϑ_2. A substantial simplification at this stage can be attained by introducing the following assumption.

Assumption 2. The thermal flux from the lamp to the active element P_{12} is much smaller than the thermal flux transmitted from the lamp to the illuminator P_{13}. This assumption is based on the premise of isotropic propagation of the thermal flux from the lamp in the radial direction and the smallness of the exposure factor φ_{12}. It was shown in [2, 3] that both assumptions are satisfactorily fulfilled in consequence of the conditions $P_1/P_2 \gtrsim 10$; $H/d > 2$ realized in real systems. The given assumption leads to the inequality $P_{12}=\sigma_{12}(\vartheta_1-\vartheta_2) \ll \sigma_{13}(\vartheta_1-\vartheta_3)=P_{13}$, which makes it possible to neglect the terms containing the cofactor σ_{12} in Eq. (1). Numerical evaluations showed that such a simplification leads to an error of less than 5% in determining the steady-state values of ϑ_1 and ϑ_2.

When the above calculation methods are used, a solution is obtained in the form

$$\left.\begin{aligned}
\vartheta_1 &= \left(\frac{P_1}{\sigma_{13}}+\vartheta_3\right)[1-\exp(-m_1\tau)],\\
\vartheta_2 &= \frac{1}{1+s_2}\left(\frac{P_2}{\sigma_{23}}+\vartheta_3+s_2\vartheta_1\right)\{1-\exp[-(1+s_2)\,m_2\tau]\},\\
\vartheta_3 &= \frac{P_3+(P_1+P_2)[1-\exp(-m_E\tau)]}{\sigma_{3c}n}[1-\exp(-nm_3\tau)],
\end{aligned}\right\} \tag{3}$$

where $m_1=\sigma_{13}/C_1$; $s_2=\sigma_{12}/\sigma_{23}$; $m_2=\sigma_{23}/C_2$; $m_E=(\sigma_{13}+\sigma_{23})/(C_1+C_2)$; $n=1+\varkappa_\Sigma\exp(-m_E\tau)$.

The compact form of the obtained relationships makes it possible to carry out the analysis of the thermal regime of the lighting system without a computer. The accuracy of the

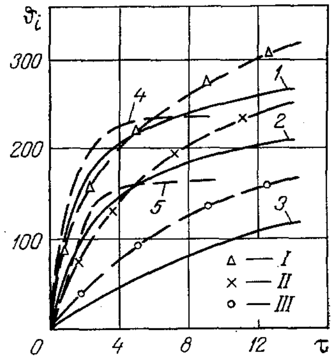

Fig. 3. Dependence of the superheating of the optical elements ϑ_i (°K) on the time of operation of the laser τ (min) with mean pumping power P = 25 W. Calculation: 1, 2, 3) superheating of lamp ϑ_1, of the active element ϑ_2, and of the illuminator; 4) ϑ_1 for $\vartheta_3 = 0$; 5) ϑ_2 for $\vartheta_3 = 0$. Experiment: I) ϑ_1; II) ϑ_2; III) ϑ_3.

calculation was verified by comparing the calculated temperatures with the experimentally measured ones.

1. Steady-State Thermal Regime. Figure 1 shows the result of calculating the dependence of ϑ_i on the mean pumping power P, carried out for a lighting system consisting of a lamp ISP 600 with bulb size between electrodes 8-mm diameter × 80 mm, an active element of neodymium glass GLS 22, 5 mm diameter × 90 mm, and an illuminator. The distance between the axes of the lamp and of the active element was 9 mm. The illuminator was of quartz and had applied to its outer surface a reflecting coating with protective layer. Heat capacities (J/°K): $C_1 = C_2 = 4$, $C_3 = 70$. The thermal losses $\eta_i = P_i/P$ were taken equal to [1, 2]: $\eta_1 = 0.5$; $\eta_2 = 0.06$; $\eta_3 = 0.25$; the remainder of the pumping power was expended on losses in the electrical circuit [1, 7], on resonator losses, and useful radiation. The range of changes of P in the calculation was chosen from the condition that the maximum temperature of the active medium had to be close to the limit temperature at which generation is possible (for $AIG:Nd^{3+}$ in a confocal resonator of mirrors with mirror ratios of ∿100%, the generation threshold was found to be ∿900°C [8]). The experimental values of ϑ_i at P = 25 W, measured with thermocouples, are also shown in Fig. 1.

The values of σ_{ij} were calculated with the assumptions of [2] by known methods [9]. The conductive and convective components of σ_{ij} (except σ_{3c}) were calculated by the formulas for cylindrical interlayers [9], the radiant components were calculated according to the Stefan—Boltzmann law.

The degree of blackness of the bodies was taken equal to $\varepsilon = 0.9$ [9], the exposure factors φ_{ij} were calculated by the method of flow algebra [10] according to the value of $\varphi_{1,2}$ calculated by the method of the "taut thread" [10, 11]. In the calculation, the method of successive approximations was used. The results of calculating σ_{ij} (P) are presented in Fig. 2. It can be seen from this figure that σ_{13} and σ_{23} are practically fully determined by radiant heat exchange but that the values of σ_{12} and σ_{3c} obtain a substantial contribution from convective and conductive heat exchange. However, with increasing temperature and increasing P, the contribution of the convective and conductive components to σ_{ij} rapidly decreases, and when P > 500, the calculation can be carried out only by taking radiant heat exchange into account. This conclusion justifies the application of an approximate calculation of the convective components of σ_{ijk} at the first stage of the calculation.

Investigations of P_{ij} showed that P_{12}, P_{13}, and P_{23} are directly proportional to the pumping power P: $P_{12} = 0.06$ P; $P_{13} = 0.5$ P; $P_{23} = 0.12$ P. The obtained results confirm the correctness of the adopted assumption 2: $P_{12} \ll P_{13}$. In the investigation of the criticality of the calculated temperatures to a change in η_i it was established that the temperature levels do not change much when η_i changes within a wide temperature range under the condition that the equality $\eta_1 + \eta_2 + \eta_3 = \text{const}$ is fulfilled. Thus, when the values $\eta_2 = 0$, 0.06, and 0.15 are substituted into (3), the respective values of ϑ_2 are 230, 250, and 280 for P = 25, and 1120, 1170, and 1250 for P = 1000. These data indicate that the temperatures of the optical elements are to a large extent determined by the overall heat release in the lighting system and by heat exchange, and that they depend to a lesser extent on the distribution of thermal losses over the elements.

Of considerable interest is an estimate of how much is contributed to the heating of the active elements by such factors as the inflow of heat from the lamp and the heat-insulating effect of the heated shell, the illuminator. Figure 1 presents the dependences ϑ_2(P) calculated on the assumption that $\sigma_{12} = 0$ (curve 4) and that $\sigma_{3c} = \infty$ or $\vartheta_3 = 0$ (curve 5). It can be seen from this figure that if the inflow of heat from the lamp to the active element is eliminated, it has little effect on the magnitude of the heating ϑ_2. A somewhat greater reduction in the magnitude of ϑ_2 is attained by the thermostabilization of the illuminator. However, with increasing P this measure becomes ever less effective because of the decrease of the radiant component of the thermal conductivity σ_{23}. When the inflow of heat from the lamp is eliminated and simultaneously the illuminator is thermostated, the decrease in the heating of the active element may be substantial (especially with low pumping power, Fig. 1, curve 6).

2. Non-Steady-State Thermal Regime. A change in time of the value of ϑ_i for $P = 25$ W, calculated for the lighting system under examination, in comparison with the experiment is presented in Fig. 3. It can be seen from the figure that, regardless of a number of substantial simplifications introduced in formulating and solving the mathematical problem, the calculation corresponds to the experiment with an accuracy that suffices for practical evaluations. The temperature increase is faster than the experimental data indicate, and this is apparently due to the fact that the calculation does not take into account the dependence of σ_{ij} on the temperature (linear approximation).

As was to be expected, in non-steady-state thermal regime there are a number of specific features due to which the conclusions obtained for steady-state regime are not applicable in the analysis of the transition regime. Figure 3 shows the results of calculating ϑ_1 and ϑ_2 for $\vartheta_3 = 0$ (curves 4 and 5). It can be seen from the figure that at the initial stage of heating, the temperatures of the elements in the thermostated illuminator may exceed the temperatures obtained with natural cooling of the illuminator; this is due to the considerable reduction of the thermal inertia of bodies 1 and 2 when $\vartheta_3 = 0$.

The obtained results make it possible to determine the factors that have the greatest effect on the thermal regime of a laser, and the directions in which further detailed research should be carried out. It was demonstrated that the temperature level of the elements of the lighting system is determined by the total heat release in the elements and by heat exchange. The heating of the active element (the most important element of the laser) depends only weakly on the nature of the distribution of the thermal losses over the individual elements, and also on the ratio of σ_{ij} (there may be some exceptions to this rule at the beginning of operation). However, if we view the obtained relationships as the initial data for formulating the boundary conditions in solving the problem of the temperature field of the active element, then it becomes imperative to determine the accurate values of η_i and σ_{ij}. This is due to the criticality of the magnitudes of the heat fluxes P_{ij} to a change in the values η_i and σ_{ij} that was revealed in the calculation. Especially notable is the role of the convective and conductive components of the conductivities. They contribute substantially to the values σ_{12} and σ_{3c} in slight heating and with low values of the mean pumping power. The convective component σ_{3c} is easily determined because calculation methods [9] are already available. Calculation of the accurate value of σ_{12k} presents much greater difficulties. Since the contribution of σ_{12k} to the value of σ_{12} is large, even if we adopt the convection factor $\varepsilon_k = 1$, it is necessary to work out a method of calculating σ_{12k}, especially for solving the problem of the temperature field of the active element. The calculation of the temperature distribution throughout the bulk of the element also requires that the relationships (3) be worked out more accurately, especially on the basis of taking into account the heat exchange with the environment on the end faces of the elements.

NOTATION

$\vartheta_i = t_i - t_c$; t_i, t_c, temperature of the i-th body and of the environment, respectively; C_i, full heat capacity of the i-th body; P_i, power of heat release in the i-th body; σ_{ij}, thermal conductivity between bodies i and j. Subscripts i, j: 1, pumping lamp, 2, active element, 3, illuminator; τ, current time; H, distance between the axes of the lamp and of the active element.

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RELAXATION OF TURBULENT STRESS

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Equations for the pulsational components of the velocity and temperature yield relaxational formulas for the turbulent stress and the heat flux.

1. Relaxation of Turbulent Stress

Attention was first drawn to the analogy between a turbulent flow of Newtonian fluid and a laminar flow of non-Newtonian fluid in [1]. There have been a number of works in which turbulence phenomena are identified with viscoelasticity phenomena [2-4]. In [5], attention turned to the possibility of making use of the pulsational components of the velocity to close the equations for the mean velocities.

From these equations, by well-known means, it is possible to write relaxational formulas for the turbulent stress. To this end, consider the general equations of motion of an incompressible liquid in the absence of external bulk forces

$$\frac{\rho \partial u_i}{\partial t} = -\frac{\partial}{\partial x_k}(p_{ki} + \rho u_i u_k), \tag{1}$$

$$\frac{\partial u_k}{\partial x_k} = 0,\ i = 1,\ 2,\ 3,\ k = 1,\ 2,\ 3. \tag{2}$$

According to Reynolds' concepts, the hydrodynamic quantities are separated into mean and pulsational components

$$u_i = \bar{u}_i + u_i',\ u_k = \bar{u}_k + u_k',\ p_{ki} = \bar{p}_{ki} + p_{ki}'. \tag{3}$$

Now the following equations for the mean and pulsational velocities may be derived from Eq. (1) [6]

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